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Published in:
Annals of Physics

DOI:
[10.1016/0003-4916\(90\)90381-W](https://doi.org/10.1016/0003-4916(90)90381-W)

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

Publication date:
1990

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Bergshoeff, E., Sezgin, E., Tani, Y., & Townsend, P. K. (1990). Super p-Branes as Gauge Theories of Volume Preserving Diffeomorphisms. *Annals of Physics*, 199(2). [https://doi.org/10.1016/0003-4916\(90\)90381-W](https://doi.org/10.1016/0003-4916(90)90381-W)

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Super p -Branes as Gauge Theories of Volume Preserving Diffeomorphisms

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Received December 21, 1988

We obtain the light-cone gauge-fixed action for a super p -brane. For $p = 2$ it is known that the action is equivalent to that of a one-dimensional super-Yang–Mills theory of the (infinite dimensional) area preserving diffeomorphism group of the membrane. We show that for $p > 2$ the action is that of a new kind of supersymmetric gauge theory of p -volume-preserving diffeomorphisms that is *not* of Yang–Mills type, and we conjecture that it is related to an infinite-dimensional non-Abelian antisymmetric tensor gauge theory. We compute the classical algebra of the supersymmetry charges and generators of the volume preserving diffeomorphisms, and show that it does not allow central extensions. © 1990 Academic Press, Inc.

1. INTRODUCTION

In discussions of gauge theories, general relativity, and string theory, the choice of light-cone gauge is often convenient because it allows the elimination of all unphysical degrees of freedom. Quantization of these theories in the light-cone gauge is then fairly straightforward because unitarity is guaranteed. The price paid

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for this is that Lorentz invariance is not manifest and may be lost upon quantization. Indeed, it *is* lost in string theory unless additional degrees of freedom are introduced or the spacetime dimension d takes one of several possible “critical” values.

The role of the light-cone gauge in the theory of membranes is rather different. It does *not* allow an elimination of all unphysical degrees of freedom. The reason for this is easy to understand. To go to the light-cone gauge we make the split,

$$X^\mu \rightarrow \left\{ X^\pm = \frac{1}{\sqrt{2}} (X^0 \pm X^{d-1}); \quad X^I, I = 1, \dots, (d-2) \right\}, \quad (1.1)$$

of the spacetime coordinates X^μ and then set $\dot{X}^+ = p^+(\tau)$ where the dot indicates differentiation with respect to the time coordinate τ on the membrane worldvolume, and p^+ is an arbitrary function of τ . As for the string, one can then solve for X^- , with the result that only the $d-2$ variables X^I remain. For the string this is the correct number of physical variables, once account is taken of invariance of the action under worldsheet diffeomorphisms. For the membrane, however, invariance under the *three-dimensional* worldvolume diffeomorphisms implies that only $d-3$ of the $d-2$ variables X^I can be physical. Thus, the light-cone gauge for the membrane must leave a residual gauge invariance [1] that would allow us, in principle, to eliminate one more variable. The action turns out to be

$$S = \frac{1}{2} \int d\tau \int d^2\sigma [(\mathcal{D}_0 X^I)^2 - \det(\partial_a X^I \partial_b X^I)], \quad (1.2)$$

where

$$\begin{aligned} \mathcal{D}_0 &= \partial_0 + u^a(\sigma, \tau) \partial_a \\ \left(\partial_0 &= \frac{\partial}{\partial \tau}, \partial_a = \frac{\partial}{\partial \sigma^a}, a = 1, 2 \right) \end{aligned} \quad (1.3)$$

is a “covariant time derivative” with the “gauge field” u^a required to satisfy

$$\partial_a u^a = 0. \quad (1.4)$$

By virtue of this gauge field, the action (1.2) possesses a gauge invariance of precisely the type to ensure that only $d-3$ of the $d-2$ variables X^I are physical.

For a membrane of spherical topology, one can solve (1.4) by writing

$$u^a = \varepsilon^{ab} \partial_b \omega. \quad (1.5)$$

By introducing the Lie bracket

$$\{f, g\}_{\text{LB}} \equiv \varepsilon^{ab} \partial_a f \partial_b g \quad (1.6)$$

for any two differentiable functions f and g on the membrane, we can then rewrite (1.2) in the suggestive form [2]

$$S = \frac{1}{2} \int d\tau \int d^2\sigma [(\partial_0 X^I - \{\omega, X^I\}_{\text{LB}})^2 - \frac{1}{2} \{X^I, X^J\}_{\text{LB}} \{X^I, X^J\}_{\text{LB}}]. \quad (1.7)$$

This has the same structure as a $(d-1)$ -dimensional Yang-Mills theory dimensionally reduced to one dimension (time). If $A^\mu = (A^0, A^I)$ and tr denotes a trace of a matrix in the adjoint representation of the Yang-Mills gauge group then the correspondence

$$\begin{aligned} X^I &\leftrightarrow A^I \\ \omega &\leftrightarrow A^0 \\ \{, \}_{\text{LB}} &\leftrightarrow [,] \\ \int d^2\sigma &\leftrightarrow \text{tr} \end{aligned} \quad (1.8)$$

makes it clear that the (non zero-mode part of the) action (1.7) is just that of a one-dimensional Yang-Mills theory with infinite dimensional gauge group [1]. It is not difficult to show that this group is the subgroup of the diffeomorphism group of the membrane (for fixed time) that preserves the Lie bracket (1.6). This bracket gives the membrane a symplectic structure, and those diffeomorphisms that preserve it are called "symplectic diffeomorphisms" or "area-preserving diffeomorphisms." The membrane model (1.7) is therefore a gauge theory of the group of symplectic diffeomorphisms, which is clearly infinite-dimensional.

The nature of this infinite-dimensional gauge group depends critically on the topology of the membrane. For spherical topology it was shown to be $SU(\infty)$, more precisely $SU_+(\infty)$,¹ by Hoppe [1]. For a toroidal membrane a basis for the algebra of symplectic diffeomorphisms was given by Floratos and Iliopoulos [3]. In this case, however, as for membranes of higher topology, one must take into account that Eq. (1.5) for u^a is valid only locally because of the non-trivial first homology group of a surface of genus $g \geq 1$.

These results were generalized to the supermembrane [4] by de Wit, Hoppe, and Nicolai [2]. The action (1.7) is thereby extended to a supersymmetric Yang-Mills theory of symplectic diffeomorphisms. In fact, starting from super-Yang-Mills theories in dimensions 3, 4, 6, and 10 with gauge group G , a dimensional reduction to one dimension (time) yields supersymmetric gauge theories with $N = 2, 4, 8$, and 16 supersymmetries, respectively. If G is taken to be the group of symplectic diffeomorphisms of a two-dimensional surface, one obtains the action of the supermembrane for $d = 4, 5, 7, 11$. These are precisely the dimensions for which the supermembrane exists. There is, therefore, a very close connection between super-

¹ J. Thierry-Mieg, private communication.

symmetric Yang–Mills theory and the supermembrane. This connection was utilized in [2] in the analysis of whether the quantized 11-dimensional supermembrane has massless particles in its spectrum, and more recently by Pope and Stelle [5] who argue that there *are* massless particles (thereby substantiating an earlier claim [6]).

The purpose of this paper is to further generalize these results to $p > 2$. We have several motivations for this. First, “supersymmetric gauge quantum-mechanical models”, or SGQM models, as such one-dimensional gauge theories are called in the quantum context, have been completely classified in the case of finite-dimensional gauge groups by Flume [7] and Baake *et al.* [8]. All of their models were of the Yang–Mills type discussed above that are related to supermembranes. It is known [9, 4, 10], however, that there are four super p -brane actions for $p > 2$, and these are presumably equivalent to SGQM models of a different type, with an infinite-dimensional gauge group. Our aim is to elucidate the nature of these models.

Second, just as for strings and membranes, the quantization of super p -branes for $p > 2$ may be expected to be greatly simplified by starting from the light-cone gauge fixed form of the action. For a spacetime of topology $(\text{Minkowski})_{d-1} \times S^1$, Bars and Pope [11] have shown that all super p -branes for $p > 2$ are anomalous, but the question remains as to whether this will be true more generally and there may yet be something to be learned from the quantum mechanics of these models.

The plan of this paper is as follows. In Section 2 we give the covariant super p -brane action and its symmetries. In Section 3, we define the light-cone gauge and analyse the field equations in this gauge. Section 4 contains the main result of the paper, which is the light-cone action for the super p -brane; the Euler–Lagrange equations of this action yield the equations in the light-cone gauge obtained in Section 3. The bosonic part of this action (omitting a zero-mode term) is

$$\frac{1}{2} \int d\tau \int d^p \sigma ([\dot{X}^I + (\partial_b \omega^{ab}) \partial_a X^I]^2 - \det(\partial_a X^I \partial_b X^I)), \quad (1.9)$$

where ω^{ab} is the gauge field of volume preserving diffeomorphisms (SDiff) of the p -brane. For $p = 2$ we recover the action (1.2) for the membrane, with its Yang–Mills type of gauge invariance. For $p > 2$ the gauge invariance of the action (1.9) is not of Yang–Mills type. The supersymmetric generalization of (1.9), and details of its gauge invariance and other symmetries can be found in Section 4. In Section 5 we construct the supercharges and the generators of SDiff, and obtain the algebra of their Dirac brackets. We show that this algebra does not allow central extensions. In Section 6, we speculate on the possibility that for $p > 2$ the action (1.9) and its supersymmetric generalizations are related to non-Abelian antisymmetric tensor gauge theories. For the convenience of the reader we give in the Appendix some technical details of the super p -brane actions and their symmetries for $p = 1, \dots, 5$.

2. THE COVARIANT SUPER p -BRANE ACTION

Our starting point will be the action [4]

$$S = - \int d^{p+1}\xi \left\{ \sqrt{-\det(\Pi_i^\mu \Pi_j^\nu \eta_{\mu\nu})} + \frac{2}{(p+1)!} \varepsilon^{i_1 \dots i_{p+1}} B_{i_1 \dots i_{p+1}} \right\} \quad (2.1)$$

which is an integral over the worldvolume (local coordinates ξ^i) of a p -dimensional extended object moving through superspace (local coordinates $Z^M = (X^\mu, \theta^\alpha)$, $\mu = 0, 1, \dots, (d-1)$). For many purposes the form of the action²,

$$S = - \frac{1}{2} \int d^{p+1}\xi \left\{ \sqrt{-g} [g^{ii} \Pi_i^\mu \Pi_j^\nu \eta_{\mu\nu} - (p-1)] + \frac{4}{(p+1)!} \varepsilon^{i_1 \dots i_{p+1}} B_{i_1 \dots i_{p+1}} \right\} \quad (2.2)$$

with g_{ij} an *independent* worldvolume metric, is a convenient alternative action. The equation for g_{ij} following from (2.2) is

$$g_{ij} = \Pi_i^\mu \Pi_j^\nu \eta_{\mu\nu} \quad (2.3)$$

and substitution of this into (2.2) yields (2.1).

In the above actions, $\eta_{\mu\nu}$ is the metric of d -dimensional Minkowski spacetime, and³

$$\Pi_i^\mu = \partial_i X^\mu - i\bar{\theta} \Gamma^\mu \partial_i \theta \quad (2.4)$$

are the components of the pullback to the worldvolume of the supersymmetric invariant one-form $\Pi^\mu = dX^\mu - i\bar{\theta} \Gamma^\mu d\theta$. The infinitesimal rigid supersymmetry transformations are

$$\delta X^\mu = i\bar{\varepsilon} \Gamma^\mu \theta, \quad \delta \theta = \varepsilon. \quad (2.5)$$

The coefficients $B_{i_1 \dots i_{p+1}}$ are given by [13]⁴

$$B_{i_1 \dots i_{p+1}} = \frac{\zeta^{-1}}{2} (i\partial_{i_{p+1}} \bar{\theta} \Gamma_{\mu_p \dots \mu_1} \theta) \times \left[\sum_{r=0}^p (-1)^r \binom{p+1}{r+1} \Pi_{i_p}^{\mu_p} \dots \Pi_{i_{r+1}}^{\mu_{r+1}} (i\partial_{i_r} \bar{\theta} \Gamma^{\mu_r} \theta) \dots (i\partial_{i_1} \bar{\theta} \Gamma^{\mu_1} \theta) \right], \quad (2.6)$$

² For $p=2$, $d=11$, the conventions of this paper agree with those of [12], except that the B_{MNP} of [12] has been replaced by $\frac{1}{2}B_{MNP}$ here. In particular, our metric conventions are “mostly plus” for both spacetime and the worldvolume.

³ The properties of θ and Γ^μ for each super p -brane ($p=1, \dots, 5$) are spelled out in the Appendix.

⁴ For the conventions used in this paper this formula is correct unless $p=3$, in which case additional insertions are required. These are given in the Appendix. With these insertions we have for *all* super p -branes $\bar{\theta} \Gamma^\mu \partial_i \theta = -\partial_i \bar{\theta} \Gamma^\mu \theta$ and $\partial_i \bar{\theta} \Gamma^{\mu_1 \dots \mu_r} \theta = -\bar{\theta} \Gamma^{\mu_1 \dots \mu_r} \partial_i \theta$. Note also that if $C \Gamma^{\mu_1 \dots \mu_r}$ is symmetric, then it follows that $C \Gamma^{\mu_1 \dots \mu_{r+2}}$ is antisymmetric.

where

$$\zeta = (-1)^{(p-2)(p-5)/4}. \quad (2.7)$$

These coefficients can be written as

$$B_{i_1 \dots i_{p+1}} = \partial_{i_1} Z^{M_1} \dots \partial_{i_{p+1}} Z^{M_{p+1}} B_{M_{p+1} \dots M_1}, \quad (2.8)$$

where the coefficients $B_{M_{p+1} \dots M_1}$ are the components of a $(p+1)$ -form in superspace,

$$B = \frac{1}{(p+1)!} dZ^{M_1} \dots dZ^{M_{p+1}} B_{M_{p+1} \dots M_1}. \quad (2.9)$$

From B we construct the closed $(p+2)$ -form

$$H = dB = \frac{1}{(p+2)!} dZ^{M_1} \dots dZ^{M_{p+1}} H_{M_{p+1} \dots M_1}. \quad (2.10)$$

Introducing the basis $\Pi^A = (\Pi^\mu, \Pi^\alpha = d\theta^\alpha)$ of supersymmetric invariant 1-forms we can rewrite H as

$$H = \frac{1}{(p+2)!} \Pi^{A_1} \dots \Pi^{A_{p+2}} H_{A_{p+2} \dots A_1}. \quad (2.11)$$

The super-Poincaré invariance of the action requires that the coefficients of H in this basis be Lorentz-invariant tensors. In fact

$$H = \frac{i\zeta^{-1}}{2p!} \Pi^{\mu_1} \dots \Pi^{\mu_p} d\bar{\theta} \Gamma_{\mu_1 \dots \mu_p} d\theta. \quad (2.12)$$

Consistency requires $H \neq 0$ but $dH = 0$, which is possible only for certain values of (p, d) . For $p \geq 2$ there are precisely eight possible (p, d) values, which fall into four sequences called the \mathbb{R} , \mathbb{C} , \mathbb{H} , \mathbb{O} sequences [10]:

$$\begin{aligned} \mathbb{R}: & (2, 4) \\ \mathbb{C}: & (2, 5), (3, 6) \\ \mathbb{H}: & (2, 7), (3, 8), (4, 9), (5, 10) \\ \mathbb{O}: & (2, 11). \end{aligned} \quad (2.13)$$

It is convenient to introduce the matrix [9]

$$\Gamma = \frac{\zeta}{(p+1)! \sqrt{-g}} \varepsilon^{i_1 \dots i_{p+1}} \Pi_{i_1}^{\mu_1} \dots \Pi_{i_{p+1}}^{\mu_{p+1}} \Gamma_{\mu_1 \dots \mu_{p+1}} \quad (2.14)$$

which satisfies⁵

$$\Gamma^2 = 1 \quad (2.15)$$

when (2.3) is satisfied.

We introduce the notation

$$\Gamma_{i_1 \dots i_n} = \Gamma_{\mu_1 \dots \mu_n} \Pi_{i_1}^{\mu_1} \dots \Pi_{i_n}^{\mu_n}, \quad \Gamma^{i_1 \dots i_n} = g^{i_1 j_1} \dots g^{i_n j_n} \Gamma_{j_1 \dots j_n} \quad (2.16)$$

and record the following lemma:

$$(\Gamma^{i_1 \dots i_n}) \Gamma = \left[\frac{(-1)^{-n(n-1)/2} \zeta}{(p+1-n)! \sqrt{-g}} \right] \varepsilon^{i_1 \dots i_n j_1 \dots j_{(p+1-n)}} \Gamma_{j_1 \dots j_{(p+1-n)}}, \quad (2.17)$$

which is valid when (2.3) is satisfied.

The general variation of the action (2.2) is

$$\begin{aligned} \delta S = & -\frac{1}{2} \int d^{p+1} \xi \left\{ \sqrt{-g} \delta g^{ij} \left[\left(\delta_i^k \delta_j^l - \frac{1}{2} g_{ij} g^{kl} \right) \Pi_k^\mu \Pi_l^\nu \eta_{\mu\nu} + \frac{1}{2} (p-1) g_{ij} \right] \right. \\ & - 2\eta_{\mu\rho} (\delta X^\rho - i\bar{\theta} \Gamma^\rho \delta \theta) \left[\partial_i (\sqrt{-g} g^{ij} \Pi_j^\mu) \right. \\ & \left. \left. - \frac{i\zeta^{-1}}{(p-1)!} \varepsilon^{i_1 \dots i_{p-1} j k} \Pi_{i_1}^{\nu_1} \dots \Pi_{i_{p-1}}^{\nu_{p-1}} \partial_j \bar{\theta} \Gamma^\mu_{\nu_1 \dots \nu_{p-1}} \partial_k \theta \right] \right. \\ & \left. + 4i \sqrt{-g} \partial_j \bar{\theta} \left(\Gamma^j + \frac{\zeta^{-1}}{p! \sqrt{-g}} \varepsilon^{i_1 \dots i_p j} \Gamma_{i_p \dots i_1} \right) \delta \theta \right\}. \quad (2.18) \end{aligned}$$

From the δg_{ij} variation one can verify that Eq. (2.3) is indeed the g_{ij} field equation. The δX^μ and $\delta \theta$ variations yield the X^μ and θ field equations. Using (2.17) they can be written in the form

$$\partial_i (\sqrt{-g} g^{ij} \Pi_j^\mu) - i \sqrt{-g} (-1)^{-p(p+1)/2} \partial_i \bar{\theta} \Gamma^\mu \Gamma^{ij} \Gamma \partial_j \theta = 0 \quad (2.19)$$

$$\{1 - (-1)^p \Gamma\} \Gamma^i \partial_i \theta = 0. \quad (2.20)$$

We conclude this section with a discussion of the gauge invariances of the action. In addition to the worldvolume diffeomorphism invariance, the action (2.1) is also invariant under the local “ κ -transformations”

$$\delta_\kappa \theta = (1 + \Gamma) \kappa(\xi), \quad \delta X^\mu = i\bar{\theta} \Gamma^\mu \delta_\kappa \theta. \quad (2.21)$$

The simplest way to establish this invariance is to write (2.1) in the form (2.2) with

⁵ For $p > 1$. For $p = 1$, Γ should be defined as in the Appendix; the formulae to follow will then apply equally for $p = 1$.

g_{ij} given implicitly by (2.3); then δg_{ij} will be given by the chain rule in terms of δZ^M but since, in this “1.5 order formalism,”

$$\delta g_{ij} \frac{\delta S}{\delta g_{ij}} \equiv 0, \quad (2.22)$$

we need not compute δg_{ij} .⁶ For a variation of the form (2.21) and using (2.17) we find that

$$\delta_\kappa S = -2i \int d^{p+1}\xi \partial_j \bar{\theta} \Gamma^j (1 - \Gamma)(1 + \Gamma) \kappa = 0 \quad (2.23)$$

3. LICHT-CONE GAUGE FIXING

In this section we will define the light-cone gauge and analyse the equations of motion for *all* the fields in this gauge. In particular, we will present the field equations which determine X^- in terms of the variables not eliminated by the gauge conditions.⁷ In the next section we shall find an action which yields the correct field equations for the latter variables.

It will prove convenient to parametrize the worldvolume metric as follows

$$g_{ij} = \begin{pmatrix} \phi^{-2}(-h + h_{cd}u^c u^d) & -\phi^{-1}h_{bc}u^c \\ -\phi^{-1}h_{ac}u^c & h_{ab} \end{pmatrix}, \quad (3.1)$$

where

$$h = \det h_{ab}. \quad (3.2)$$

The variables ϕ, u^a are related to the lapse and shift functions of the Hamiltonian formulation of general relativity. The inverse metric is then

$$g^{ij} = \begin{pmatrix} -\phi^2 h^{-1} & -\phi h^{-1} u^b \\ -\phi h^{-1} u^a & h^{ab} - h^{-1} u^a u^b \end{pmatrix}, \quad (3.3)$$

where h^{ab} is the inverse of the p -metric h_{ab} . Note also that

$$\sqrt{-g} = \phi^{-1} h. \quad (3.4)$$

To pass to the light-cone gauge we set

$$X^+ = x^+(\tau), \quad \phi = 1, \quad \Gamma^+ \theta = 0 \quad (3.5)$$

⁶ For the “first order form” of the action (2.2) it is possible to find a transformation of δg_{ij} which, together with (2.21), is a symmetry. The variation of the independent field g_{ij} in this formulation will differ from its variation as a dependent variable in the Dirac form (2.1) by field equation terms.

⁷ These equations are relevant for the construction of the non-linearly realized Lorentz generators.

in the equations of motion (2.3), (2.19), (2.20), where

$$X^\pm = \frac{1}{\sqrt{2}} (X^0 \pm X^{d-1}), \quad \Gamma^\pm = \frac{1}{\sqrt{2}} (\Gamma^0 \pm \Gamma^{d-1}) \quad (3.6)$$

and $x^+(\tau)$ is an arbitrary function of τ . In this gauge,

$$\begin{aligned} \Pi_0^+ &= \dot{x}^+(\tau) \equiv p^+(\tau), & \Pi_0^- &= \dot{X}^- - i\theta \Gamma^- \partial, & \Pi_0^I &= \dot{X}^I, \\ \Pi_a^+ &= 0, & \Pi_a^- &= \partial_a X^- - i\theta \Gamma^- \partial_a \theta, & \Pi_a^I &= \partial_a X^I \end{aligned} \quad (3.7)$$

and any bilinear in θ must involve a Γ^- if it is not to vanish.

It will prove convenient to introduce the matrix

$$\tilde{\Gamma} = \frac{\eta}{p!} \varepsilon^{a_1 \dots a_p} \partial_{a_1} X^{I_1} \dots \partial_{a_p} X^{I_p} \Gamma_{I_1 \dots I_p} \quad (3.8)$$

which is an analogue of the matrix Γ of the previous section. If we choose

$$\eta = (-1)^{p(p-1)/4} \quad (3.9)$$

then

$$\tilde{\Gamma}^2 = h \quad (3.10)$$

when the h_{ab} field equation (given in (3.14a) below) is satisfied. In the light-cone gauge, Γ can be written in terms of $\tilde{\Gamma}$ as follows:

$$\begin{aligned} \Gamma &= \frac{(\zeta \eta^{-1})}{2h} [\Gamma_I \mathcal{D}_0 X^I \tilde{\Gamma} \Gamma^+ + 2(-1)^p p^+ \tilde{\Gamma}] \Gamma^- \\ &+ \frac{(-1)^{-(p+1)/2}}{2h} \left[\frac{h}{p^+} \tilde{\Gamma} - \tilde{\Gamma} \Gamma_I \mathcal{D}_0 X^I \Gamma^- - \frac{1}{p^+} (\mathcal{D}_0 X^I)^2 \tilde{\Gamma} \right. \\ &\left. + \frac{2}{p^+} \mathcal{D}_0 X^I \partial_a X^J \mathcal{D}_0 X^K \partial^a X^K \Gamma_I \Gamma_J \tilde{\Gamma} \right] \Gamma^+. \end{aligned} \quad (3.11)$$

As in the previous section we define

$$\begin{aligned} \tilde{\Gamma}_{a_1 \dots a_n} &= \partial_{a_1} X^{I_1} \dots \partial_{a_n} X^{I_n} \Gamma_{I_1 \dots I_n} \\ \tilde{\Gamma}^{a_1 \dots a_n} &= h^{a_1 b_1} \dots h^{a_n b_n} \tilde{\Gamma}_{b_1 \dots b_n}. \end{aligned} \quad (3.12)$$

Given that $h_{ab} = \partial_a X^I \partial_b X^I$ these matrices satisfy the relation

$$(\tilde{\Gamma}^{a_1 \dots a_n}) \tilde{\Gamma} = \frac{(-1)^{n(n-1)/2}}{(p-n)!} \varepsilon^{a_1 \dots a_n b_1 \dots b_{(p-n)}} \tilde{\Gamma}_{b_1 \dots b_{(p-n)}} \quad (3.13)$$

which is analogous to (2.17).

Our task now is to rewrite the equations of motion (2.3), (2.19), and (2.20) that follow from the action (2.2) using the light-cone gauge conditions. Starting with (2.3) we find that this produces the three equations

$$h_{ab} = \partial_a X^I \partial_b X^I \quad (3.14a)$$

$$\partial_a X^- - i\bar{\theta}\Gamma^- \partial_a \theta - \frac{1}{p^+} \mathcal{D}_0 X^I \partial_a X^I = 0 \quad (3.14b)$$

$$\mathcal{D}_0 X^- - i\bar{\theta}\Gamma^- \mathcal{D}_0 \theta - \frac{1}{2p^+} [(\mathcal{D}_0 X^I)^2 + h] = 0, \quad (3.14c)$$

where $\{X^I, I = 1, \dots, (d-2)\}$ are the “transverse” X -coordinates, and

$$\mathcal{D}_0 \Phi = (\partial_0 + u^a \partial_a) \Phi \quad (3.15)$$

for any p -brane scalar Φ . The derivative \mathcal{D}_0 will play the role of a covariant derivative in what follows. We shall use Eq. (3.14a) in all subsequent equations, so that *whenever h_{ab} appears it should be thought of as shorthand for $\partial_a X^I \partial_b X^I$* . In particular, this means that we may freely use the Eqs. (3.10) and (3.13).

From (3.14b) we can solve for $X^-(\tau, \sigma)$ except for its average $x^-(\tau)$, equal to $(\int d^p \sigma X^- / \int d^p \sigma)$, which is a variable that should *not* be eliminated after the light-cone gauge choice. (See, for example, [14] for a discussion of this point in the case of string theories).

Integration of (3.14b) around any cycle on the p -brane yields the constraint

$$\oint \mathcal{F} = 0, \quad (3.16)$$

where \mathcal{F} is the one-form

$$\Phi = d\sigma^a (\mathcal{D}_0 X^I \partial_a X^I + ip^+ \bar{\theta} \Gamma^- \partial_a \theta). \quad (3.17)$$

This constraint requires \mathcal{F} to be exact. It follows that \mathcal{F} is closed, $d\mathcal{F} = 0$, which is equivalent to the *local* constraint

$$\partial_{[a} \mathcal{D}_0 X^I \partial_{b]} X^I + ip^+ \partial_{[a} \bar{\theta} \Gamma^{-1} \partial_{b]} \theta = 0. \quad (3.18)$$

However, if the p -brane configuration is such that the first homology group is non-trivial there will exist *harmonic* 1-forms \mathcal{F} , and on integration over non-trivial 1-cycles these will yield additional *global* constraints. Note that for the closed string the local constraint (3.18) is trivial but there is one global constraint obtained by integrating \mathcal{F} around the closed loop formed by the string.

Equation (3.14c) contains no further information. We show after Eq. (3.26) that it follows from (3.14b) and the field equations for X^I and θ . This redundancy of information is of course a consequence of the gauge invariance of the original action.

We next consider the X^μ equation (2.19). For $\mu = +$, this yields the X^- equation

$$\partial_a u^a + \frac{\dot{p}^+}{p^+} = 0. \quad (3.19)$$

Integrating this over the closed p -brane we find that

$$\dot{p}^+ = 0 \quad (3.20)$$

which is the field equation of $x^-(\tau)$. It then follows that

$$\partial_a u^a = 0. \quad (3.21)$$

This equation can be solved by writing u^a as

$$u^a = \partial_b \omega^{ab} + \sum_{r=1}^{b_1} \frac{1}{\sqrt{h}} u_H^{(r)a}, \quad (3.22)$$

where ω^{ab} will turn out to be the Lagrange multiplier for the local constraint (3.18) and $u_H^{(r)a}$, equal in number to the first Betti number b_1 of the p -brane, are harmonic vectors that will be the Lagrange multipliers for the global constraints.

For $\mu = -$ we obtain the X^+ equation, which becomes, on using (3.14b) and (3.21),

$$\begin{aligned} \mathcal{D}_0^2 X^- - i\bar{\theta}\Gamma^- \mathcal{D}_0^2 \theta + \frac{1}{p^+} \partial_a (hh^{ab} \mathcal{D}_0 X^I \partial_b X^I) \\ - i(\zeta\eta)^{-1} \mathcal{D}_0 X^I \partial_a \bar{\theta} \Gamma^- \Gamma^I \tilde{F}^{ab} \tilde{F} \partial_b \theta = 0. \end{aligned} \quad (3.23)$$

This is the time derivative of the ϕ equation (3.14c) and so, like the latter, yields no additional information.

For $\mu = I$ we obtain

$$\mathcal{D}_0^2 X^I - \partial_a (hh^{ab} \partial_b X^I) + i(\zeta\eta)^{-1} p^+ \partial_a \bar{\theta} \Gamma^- \Gamma^I \tilde{F}^{ab} \tilde{F} \partial_b \theta = 0. \quad (3.24)$$

Finally we turn to the θ -equation (2.20). In the light-cone gauge this becomes

$$\begin{aligned} \Gamma^- \{1 - (-1)^p \Gamma\} \mathcal{D}_0 \theta - \frac{1}{2(p^+)^2} [(\mathcal{D}_0 X^I)^2 + h] \Gamma^+ \{1 - (-1)^p \Gamma\} \mathcal{D}_0 \theta \\ + \frac{1}{p^+} \mathcal{D}_0 X^I \Gamma^I \{1 - (-1)^p \Gamma\} \mathcal{D}_0 \theta + \frac{1}{(p^+)^2} hh^{ab} \mathcal{D}_0 X^I \partial_a X^I \Gamma^+ \{1 - (-1)^p \Gamma\} \partial_b \theta \\ - \frac{1}{p^+} hh^{ab} \partial_a X^I \Gamma^I \{1 - (-1)^p \Gamma\} \partial_b \theta = 0. \end{aligned} \quad (3.25)$$

Substituting for Γ from (3.11) and multiplying by Γ^+ we directly obtain

$$\mathcal{D}_0(\Gamma^-\theta) + \left(\frac{\zeta\eta^{-1}}{p^+}\right) \tilde{F}^a \tilde{F} \partial_a(\Gamma^-\theta) = 0. \quad (3.26)$$

The equation obtained by multiplying (3.25) by Γ^- is then identically satisfied upon use of (3.26). Again, this is a consequence of the gauge invariance of the original action.

As promised, we now verify the consistency of (3.14b) and (3.14c). To this end it is convenient to extend the definition of \mathcal{D}_0 to act on worldvolume *vectors*. Thus

$$\mathcal{D}_0 V_a = (\partial_0 + u^b \partial_b) V_a + (\partial_a u^b) V_b. \quad (3.27)$$

It follows that

$$[\mathcal{D}_0, \partial_a] \Phi = 0 \quad (3.28)$$

for a scalar Φ . Acting now with \mathcal{D}_0 on (3.14b) we recover ∂_a of (3.14c) on using the X' and θ equations of motion. Further acting with \mathcal{D}_0 on (3.14c) we recover (3.23) upon use of (3.14b) and the X' equation.

We now simplify the independent equations by choosing the Γ -matrix representation such that⁸

$$\Gamma^-\theta = (\sqrt{2}p^+)^{-1/2} \begin{pmatrix} 0 \\ S \end{pmatrix} \quad (3.29)$$

where S is a spinor of $SO(d-2)$. In this representation we have⁹

$$\Gamma^I = \gamma^I \otimes \sigma_3, \quad \Gamma^+ = \mathbb{1} \otimes \begin{pmatrix} 0 & 0 \\ \sqrt{2}i & 0 \end{pmatrix}, \quad \Gamma^- = \mathbb{1} \otimes \begin{pmatrix} 0 & -i\sqrt{2} \\ 0 & 0 \end{pmatrix} \quad (3.30)$$

where γ^I are reduced γ -matrices that satisfy

$$\{\gamma^I, \gamma^J\} = 2\delta^{IJ}, \quad \text{tr}(\gamma^I \gamma^J) = (d-p-1) \delta^{IJ}. \quad (3.31)$$

We can write \tilde{F} as

$$\tilde{F} = \gamma \otimes (\sigma_3)^p, \quad (3.32)$$

where

$$\gamma = \frac{\eta}{p!} e^{a_1 \dots a_p} \partial_{a_1} X^{I_1} \dots \partial_{a_p} X^{I_p} \gamma_{I_1 \dots I_p} \quad (3.33)$$

⁸ The Γ -matrices defined in the Appendix can be chosen to be either real or pure imaginary. In the latter case an additional factor of i is required in this equation in order that the spinor S be real, as is assumed in the following analysis.

⁹ This corrects the corresponding formula in [12].

also satisfies $\gamma^2 = h$. We define

$$\gamma_{a_1 \dots a_n} = \partial_{a_1} X^{I_1} \dots \partial_{a_n} X^{I_n} \gamma_{I_1 \dots I_n}. \quad (3.34)$$

In terms of X^I and S , the local constraint (3.18) and the equations of motion (3.24) and (3.26) read¹⁰

$$L_{ab} \equiv \partial_{[a} \mathcal{D}_0 X^I \partial_{b]} X^I + \frac{i}{2} \partial_{[a} \bar{S} \partial_{b]} S = 0 \quad (3.35a)$$

$$B^I \equiv -\mathcal{D}_0^2 X^I + \partial_a (h h^{ab} \partial_b X^I) + \frac{i^p}{2} \partial_a \bar{S} (\gamma^I \gamma^{ab} - \gamma^{abc} \partial_c X^I) \gamma \partial_b S = 0 \quad (3.35b)$$

$$F \equiv \mathcal{D}_0 S + i^{p+1} \gamma^a \gamma \partial_a S = 0, \quad (3.35c)$$

where $\bar{S} = S^T$. These can be considered as the field equations of a “light-cone action” involving the variables ω^{ab} (defined in (3.22)), X^I and S , respectively. In the next section we shall give this action and discuss its symmetries. We remark here that, since $\int d^p \sigma L_{ab} \equiv 0$, there is no symmetry of the action corresponding to the *constant* mode of $L_{ab}(\sigma)$.

4. THE LIGHT-CONE ACTION AND ITS SYMMETRIES

In the previous section we have obtained the equations of motion of the super p -brane action (3.2) in the light-cone gauge. Ignoring the *global* constraints, the remaining variables are ω^{ab} , X^I , S , $x^+(\tau)$, and $x^-(\tau)$. We now seek an action for these variables that yields their equations of motion. It is not difficult to show that

$$I = I_0 + \frac{1}{2} \int d^{p+1} \xi \{ (\mathcal{D}_0 X^I)^2 + i \bar{S} \mathcal{D}_0 S - h - i^p \bar{S} \gamma^a \gamma \partial_a S \}, \quad (4.1)$$

where

$$I_0 = \int d\tau \dot{x}^+ \dot{x}^-, \quad (4.2)$$

is such an action. We recall that

$$\mathcal{D}_0 X^I = \dot{X}^I + (\partial_b \omega^{ab}) \partial_b X^I. \quad (4.3)$$

Note that, the x^- field equation is $\ddot{x}^- = 0$, i.e., $\dot{p}^+ = 0$, in agreement with (3.20). Likewise the x^+ field equation is $\ddot{x}^+ = 0$, which is in agreement with (3.23); these equations express the fact the Hamiltonian of the system, which is $p^+ \dot{x}^- \equiv p^+ p^-$, is time-independent.

¹⁰ These formulae need a modification for $p = 3$, as explained in the Appendix.

The action (4.1) has the following gauge invariance (for $p \geq 2$)

$$\begin{aligned}\delta X^I &= (\partial_b A^{ab}) \partial_a X^I \\ \delta S &= (\partial_b A^{ab}) \partial_a S \\ \delta \omega^{ab} &= -\mathcal{D}_0 A^{ab},\end{aligned}\tag{4.4}$$

where $A^{ab} = -A^{ba}$, and \mathcal{D}_0 acts on the tensor A^{ab} by extension of the formula for vectors given in (3.27).

For $p = 2$ we set

$$A^{ab} = \varepsilon^{ab} A, \quad \omega^{ab} = \varepsilon^{ab} \omega\tag{4.5}$$

and define the Lie bracket as in (1.6) to obtain

$$\delta X^I = -\{\omega, X^I\}_{\text{LB}}, \quad \delta S = -\{\omega, S\}_{\text{LB}}, \quad \delta \omega = -\mathcal{D}_0 A\tag{4.6}$$

which are the transformations of a $(d-1)$ dimensional supersymmetric Yang-Mills theory reduced to one dimension. Thus, as mentioned in the introduction, the $d = 4, 5, 7$, and 11 dimensional supermembrane actions are equivalent to the dimensionally reduced $d = 3, 4, 6$, and 10 dimensional super Yang-Mills theories.

For $p \geq 3$, for which there are a total of four cases, the gauge invariance of the action is *not* of Yang-Mills type. In this case, there is also the Abelian antisymmetric-tensor type of gauge transformation for ω^{ab} ;

$$\delta \omega^{ab} = \partial_c A^{abc},\tag{4.7}$$

where the parameter A^{abc} is totally antisymmetric in abc .

In addition to these gauge invariances, there are also rigid symmetries. In particular we have d -dimensional super-Poincaré invariance, of which only the $SO(d-2) \times SO(1, 1)$, and half of the supersymmetry, is linearly realized. We also have the constant worldvolume time-translations which survive the light-cone gauge choice. These are

$$\begin{aligned}\delta X^I &= v \dot{X}^I + a^I \\ \delta S &= v \dot{S} \\ \delta \omega^{ab} &= v \mathcal{D}_0 \omega^{ab},\end{aligned}\tag{4.8}$$

where, for later convenience, we have included the “transverse” rigid target (Minkowski) space translations with infinitesimal parameter a^I . On these fields (i.e., excluding x^+ , x^-) the non-linearly realized supersymmetry takes the remarkably simple form

$$\delta S = \beta,\tag{4.9}$$

where β is constant. In checking this one must use the identity

$$\partial_a(\gamma^{ab_1 \dots b_n} \gamma) = 0 \quad (4.10)$$

which follows from (3.13).

The linearly realized supersymmetry takes the form

$$\begin{aligned} \delta X^I &= -2i\bar{S}\gamma^I\alpha \\ \delta S &= -2\mathcal{D}_0 X^I \gamma^I \alpha - 2i^{-p+1} \gamma \alpha \\ \delta \omega^{ab} &= -2i^{-p} \bar{S} \gamma^{ab} \gamma \alpha. \end{aligned} \quad (4.11)$$

To show the α -invariance of the action (4.1) one needs the following identity:

$$(\gamma^{abc} \gamma)_{(\alpha\beta} (\gamma_c)_{\gamma)\delta} - (\gamma^I \gamma^{ab} \gamma)_{(\alpha\beta} (\gamma^I)_{\gamma)\delta} + (-1)^p \delta_{(\alpha\beta} (\gamma^{ab} \gamma)_{\gamma)\delta} = 0. \quad (4.12)$$

which follows from the identity [4, 10]

$$\Gamma_{(\alpha\beta}^\mu \Gamma_{\gamma\delta)}^{\mu\nu_1 \dots \nu_{(p-1)}} = 0 \quad (4.13)$$

used in the verification of the κ -invariance of the super p -brane action. In checking the invariance of the light-cone action under (4.11), it is useful to note that

$$\delta(\gamma^{a_1 \dots a_n} \gamma) = \delta(\partial_b X^I) (\gamma^I \gamma^{ba_1 \dots a_n} - \partial_c X^I \gamma^{cba_1 \dots a_n}) \gamma \quad (4.14a)$$

$$\gamma \gamma^a + (-1)^p \gamma^a \gamma = 0. \quad (4.14b)$$

The on-shell commutator algebra is

$$[\delta(\alpha_1), \delta(\alpha_2)] = \delta(v_3 = 8i\bar{\alpha}_2 \alpha_1)$$

$$+ \delta \left(A_3^{ab} = \left[-4i\omega^{ab} \bar{\alpha}_2 \alpha_1 + \frac{4i^{-p}}{1-p} X^I \bar{\alpha}_2 \gamma_I \gamma^{ab} \gamma \alpha_1 \right] - (1 \leftrightarrow 2) \right) \quad (4.15)$$

$$[\delta(\alpha), \delta(\beta)] = \delta(a_3^I = 2i\bar{\beta} \gamma^I \alpha) \quad (4.16)$$

$$[\delta(A_1), \delta(A_2)] = \delta(A_3^{ab} = \partial_c A_2^{ac} \partial_a A_1^{bd} - (1 \leftrightarrow 2)) \quad (4.17)$$

$$[\delta(v), \delta(A)] = \delta(v_3 = \partial_b A^{ab} \partial_a v) + \delta(A_3^{ab} = -v \dot{A}^{ab}). \quad (4.18)$$

This algebra is derived from the transformation rules of X^I , S , and ω^{ab} , excluding the zero modes x^+ , x^- . To extend this algebra to include all transformations of the original rigid spacetime super-Poincaré group we would have to include x^+ , x^- . Only in this way can one derive, for example, the form of the non-linear L^{I+} generator in the light-cone gauge; we shall not address this problem in this paper. Observe that A^{ab} is determined only up to a term of the form $\partial_c A^{abc}$. In fact, the constant mode in X^I leads to just such a term and so, as expected, is irrelevant.

In showing the closure on S of the supersymmetry algebra (4.15) we have used the identity

$$\begin{aligned} & [(\bar{S}\gamma^I\alpha_1)\bar{\alpha}_2\gamma^I\gamma^a\gamma + (\bar{S}\gamma_b\alpha_1)\bar{\alpha}_2\gamma^{ab}\gamma \\ & + (-1)^{p+1}(\bar{S}\gamma^I\gamma^a\gamma\alpha_1)\bar{\alpha}_2\gamma^I + (-1)^{p+1}(\bar{S}\gamma^{ab}\gamma\alpha_1)\bar{\alpha}_2\gamma_b] - (1 \leftrightarrow 2) \\ & = [(\bar{\alpha}_1\alpha_2)\bar{S}\gamma^a\gamma + (-1)^{p+1}(\bar{\alpha}_1\gamma^a\gamma\alpha_2)\bar{S}] - (1 \leftrightarrow 2) \end{aligned} \quad (4.19)$$

which follows from (4.13). The closure on ω^{ab} , on the other hand, requires the identity

$$\{[\bar{\alpha}_1\gamma^{abc}\gamma^I\gamma S_3)(\bar{S}_4\gamma^I\alpha_2) + (\bar{\alpha}_1\gamma^{abcd}\gamma S_3)(\bar{S}_4\gamma_d\alpha_2)] - (1 \leftrightarrow 2) - (3 \leftrightarrow 4) = 0 \quad (4.20)$$

which is valid whenever (4.12) holds. For gauge-invariant quantities the commutator algebra of (4.15) is precisely that of N -extended supersymmetry, where N is the number of components of α .

We conclude this section with some properties of the field equations and constraint (3.35). First, the identity

$$\mathcal{D}_0 L_{ab} + \partial_{[a} B^I \partial_{b]} X^I - \partial_{[a} \bar{F} \partial_{b]} S = 0 \quad (4.21)$$

ensures that the constraint $L_{ab} = 0$ is consistent with the time evolution. Second, under α -symmetry the equations of motion and the constraint transform into each other as follows

$$\begin{aligned} \delta B^I &= 2\mathcal{D}_0 \bar{F} \gamma^I \alpha - 2i^{-p-1} \partial_a \bar{F} (\gamma^I \gamma^a - \partial_b X^I \gamma^{ba}) \gamma \alpha \\ &\quad + 2i^{-p} \partial_a \bar{S} (\gamma^I \gamma^{abc} - \partial_d X^I \gamma^{dabc}) \gamma \alpha L_{bc} \\ \delta F &= 2i B^I \gamma_I \alpha - 2i^{-p} \gamma^{ab} \gamma \alpha L_{ab} \\ \delta L_{ab} &= 2\partial_{[a} (\bar{\alpha} \gamma_{b]} F). \end{aligned} \quad (4.22)$$

5. THE ALGEBRA OF SUPERCHARGES AND SDIFF

In this section we give, in the light-cone gauge, the generators Q^- , Q^+ , and L^{ab} which generate the α - and β -supersymmetries and SDiff, respectively. We then compute their Dirac brackets.

The supercharge

$$\begin{aligned} Q &= \frac{1}{2} \Gamma^+ \Gamma^- Q + \frac{1}{2} \Gamma^- \Gamma^+ Q \\ &\equiv Q^- + Q^+, \end{aligned} \quad (5.1)$$

can be derived as a Noether charge corresponding to the supersymmetry of the action (2.2). The standard Noether procedure yields the result

$$Q = \int d^p \sigma (i P^\mu \Gamma_\mu \theta + P + \varepsilon^{a_1 \dots a_p} \Gamma_{a_1 \dots a_p} \theta + \dots), \quad (5.2)$$

where dots represent terms that are cubic and higher order in θ , all of which vanish in the light-cone gauge, and the conjugate momenta P_μ and P are given by

$$P_\mu = \phi \Pi_{0\mu} + u^a \Pi_{a\mu} + \varepsilon^{a_1 \dots a_p} \Pi_{a_1}^{A_1} \dots \Pi_{a_p}^{A_p} B_{A_p \dots A_1 \mu} \quad (5.3)$$

$$\bar{P}_\alpha = -i(\bar{\theta} \Gamma^\mu)_\alpha P_\mu - \varepsilon^{a_1 \dots a_p} \Pi_{a_1}^{A_1} \dots \Pi_{a_p}^{A_p} B_{A_p \dots A_1 \alpha}. \quad (5.4)$$

The generators of SDiff are

$$L_{ab}(\sigma) = \partial_{[a} \mathcal{F}_{b]}(\sigma) \quad (5.5)$$

and

$$P^{(r)} = \oint_{c^{(r)}} d\sigma^a \mathcal{F}_a, \quad (5.6)$$

where

$$\mathcal{F}_a = \Pi_a^\mu (P_\mu - \varepsilon^{a_1 \dots a_p} \Pi_{a_1}^{A_1} \dots \Pi_{a_p}^{A_p} B_{A_p \dots A_1 \mu}) \quad (5.7)$$

and where $c^{(r)}$ ($r = 1 \dots b_1$) are the 1-cycle representatives of the first homology group of the p -brane, with b_1 the first Betti number.

In the light-cone gauge, defined by (3.5), we find that (we suppress the range of the definite integrals, and the normalization constant equal to $\int d^p \sigma$)

$$Q^- = \int d^p \sigma (P_I \gamma^I + i^{p-1} \gamma) S \quad (5.8)$$

$$Q^+ = \int d^p \sigma S \quad (5.9)$$

$$\begin{aligned} H &\equiv -p^+ \int d^p \sigma P^-(\tau, \sigma) \\ &= \int d^p \sigma \left(\frac{1}{2} P_I P_I + \frac{1}{2} h - \frac{i^p}{2} \bar{S} \gamma^a \gamma \partial_a S \right) \end{aligned} \quad (5.10)$$

$$\mathcal{F}_a = P_I \partial_a X^I + \frac{i}{2} \bar{S} \partial_a S, \quad (5.11)$$

where $P^I = D_0 X^I$ and we have performed the redefinitions: $Q^- \rightarrow -\sqrt{2/p^+} Q^-$, $Q^+ \rightarrow \sqrt{2p^+} Q^+$. In obtaining (5.10) we have used (3.14c). We observe that H defined in (5.10) is the generator of constant translations in X^+ (i.e., time), and it

agrees with the Hamiltonian one would obtain from $(I - I_0)$ of (4.1), prior to the addition of the first class constraint L_{ab} .

In order to compute the Dirac brackets of the generators defined above, we need to know the Dirac brackets of the canonical variables X^I and S , and their conjugate momenta. For the case of the supermembrane this was done in [15] and it is easy to see that the result of [15] generalizes to all super p -branes as follows:

$$\{X^I(\underline{\sigma}), P_J(\underline{\sigma}')\}^* = \delta^I_J(\underline{\sigma}, \underline{\sigma}') \delta^I_J \quad (5.12)$$

$$\{S^\alpha(\underline{\sigma}), \bar{S}_\beta(\underline{\sigma}')\}^* = -i\delta^\alpha_\beta(\underline{\sigma}, \underline{\sigma}') \delta^\alpha_\beta, \quad (5.13)$$

where $\delta^p(\underline{\sigma}, \underline{\sigma}')$ is the Dirac delta function.

One can now calculate the Dirac brackets of all the generators. The result for the non-vanishing Dirac brackets is

$$\{Q^{\alpha-}, \bar{Q}_\beta^-\}^* = -2i\delta^\alpha_\beta H - \frac{2i^p}{(p-1)} \int d^p \sigma (\gamma^I \gamma^{ab} \gamma)^\alpha_\beta X^I L_{ab} \quad (5.14)$$

$$\{Q^{\alpha-}, \bar{Q}_\beta^+\}^* = -i \int d^p \sigma (\gamma^I)^\alpha_\beta P^I \equiv -i(\gamma^I)^\alpha_\beta P^I_0 \quad (5.15)$$

$$\{Q^{\alpha+}, \bar{Q}_\beta^+\}^* = -i\delta^\alpha_\beta \quad (5.16)$$

$$\{H, \bar{Q}^-\}^* = i^{-p+1} \int d^p \sigma \bar{S} \gamma^{ab} \gamma L_{ab} \quad (5.17)$$

$$\{L_{ab}(\underline{\sigma}), L_{cd}(\underline{\sigma}')\}^* = \partial_c L_{ab}(\underline{\sigma}) \partial_d \delta^p(\underline{\sigma}, \underline{\sigma}') - [(ab, \underline{\sigma}) \leftrightarrow (cd, \underline{\sigma}')] \quad (5.18)$$

$$\{P^{(r)}, L_{ab}(\underline{\sigma})\}^* = -\oint_{c^{(r)}} d\sigma'^c \partial'_{[a} \delta^p(\underline{\sigma}, \underline{\sigma}') L_{b]c}(\underline{\sigma}') \quad (5.19)$$

$$\{P^{(r)}, P^{(s)}\}^* = -2 \oint_{c^{(r)}} \oint_{c^{(s)}} d\sigma^a d\sigma'^b \delta^p(\underline{\sigma}, \underline{\sigma}') L_{ab}(\underline{\sigma}). \quad (5.20)$$

We recall from the end of Section 3 that the *constant* mode of $L_{ab}(\underline{\sigma})$ vanishes identically. Therefore, the term on the right hand side of (5.14) involving the constant mode of X^I vanishes, as required.

In the derivation of (5.14), one needs the identity (4.12). On fields which are invariant under SDiff, the algebra reduces to the subalgebra of the target space super-Poincaré algebra containing the supercharges Q^\pm and the momentum P^μ . We could have included the Lorentz generators, in which case for SDiff-invariant fields the algebra would reduce to the full target space super Poincaré algebra.

We now consider the possibility of adding central extensions to the algebra (5.14)–(5.20). It has been recently shown [16, 17] that the bosonic part of this algebra, i.e., (5.18)–(5.20), admits central extensions (allowed by the Jacobi identities) which are intimately connected with the existence of harmonic 1-forms on the p -brane. The inclusion of the supercharges Q^\pm and the Hamiltonian H requires that additional Jacobi identities are satisfied. Assuming that the central extensions are

field independent, we have checked that the Jacobi identities involving the triplets (L, H, Q) and (L, Q, Q) do not allow central extensions in *any* of the Dirac brackets (5.14)–(5.20). It is not clear to us whether the impossibility of central extensions of the algebra (5.14)–(5.20) implies the absence of anomalies. In any case, it should be noted that the absence of possible central extensions in a subalgebra does not rule out their presence in the full algebra.

We conclude this section by giving an explicit form of the algebra (5.14)–(5.20) for the case of a toroidal membrane, thereby recovering the results of [2, 3]. On a flat 2-torus the Fourier components of $L(\sigma) \equiv \varepsilon^{ab} L_{ab} = \varepsilon^{ab} \partial_a \mathcal{F}_b$ are

$$L_n = \int d^2\sigma e^{in \cdot \sigma} L(\sigma), \quad n \in \mathbb{Z}, n \neq 0. \quad (5.21)$$

On a 2-torus there are two 1-cycles, and therefore there are two global generators. Choosing Cartesian coordinates x, y with the identifications $x \sim x + 1, y \sim y + 1$, and choosing $c^{(1)}$ and $c^{(2)}$ to be the circles $y = \text{const}$ and $x = \text{const}$, respectively, we have

$$P_1 = \int_0^1 dx \mathcal{F}_x(x, y), \quad P_2 = \int_0^1 dy \mathcal{F}_y(x, y). \quad (5.22)$$

Different choices of y in the expression for P_1 , and x in the expression for P_2 , lead to generators which differ by the addition of a linear combination of the $\{L_n\}$. The quantities

$$P_a = \int d^2\sigma \mathcal{F}_a \quad (5.23)$$

obtained by averaging those of (5.22), therefore constitute an equivalent set of global generators.

In terms of the generators introduced above for the 2-torus, the algebra (5.14)–(5.20), becomes

$$\begin{aligned} \{L_m, L_n\} &= i(\underline{m} \times \underline{n}) L_{\underline{m}+\underline{n}}, \quad (\underline{m}, \underline{n} \neq \underline{0}) \\ \{P_a, L_n\} &= in_a L_n \\ \{P_a, P_b\} &= 0 \\ \{Q^{\alpha-}, \bar{Q}_\beta^-\} &= 2iH\delta_\beta^\alpha + 2i \sum_n X_n^I (\gamma_I)^\alpha{}_\beta L_{-n} \\ \{Q^{\alpha-}, \bar{Q}_\beta^+\} &= -i(\gamma^I)^\alpha{}_\beta P_0^I \\ \{Q^{\alpha+}, \bar{Q}_\beta^+\} &= -i\delta_\beta^\alpha \\ \{H, Q^\alpha\} &= -\sum_n S_n L_{-n}, \end{aligned} \quad (5.24)$$

where X_n^I and S_n are the Fourier components of X^I and S , respectively.

6. COMMENTS

As mentioned in the Introduction, there is a close connection between supermembrane theories in $d=4, 5, 7$, and 11 and $N=1$ supersymmetric Yang–Mills theories $[1, 2]$ in $d=3, 4, 6$, and 10 , or equivalently $N=1, 2, 4$, and 8 supersymmetric Yang–Mills theories in $d=3$. One may wonder whether any relation exists between the higher super p -branes ($p>2$) and certain supersymmetric field theories. We conjecture that there is a connection between super p -brane theories and supersymmetric field theories in $(p+1)$ -dimensions containing a $(p-1)$ th rank antisymmetric tensor gauge field. According to this conjecture the relevant field content of these supersymmetric field theories would be as presented in the table below:

p	1	2	3	4	5
Spacetime dim. of SUSY field theory	2	3	4	5	6
The $(p-1)$ -form	A	A_μ	$A_{\mu\nu}$	$A_{\mu\nu\rho}$	$A_{\mu\nu\rho\sigma}$
Fermi + Bose deg. of freedom, and family	$R: 1+1$ $C: 2+2$ $H: 4+4$ $O: 8+8$	$1+1$ $2+2$ $4+4$ $8+8$	$2+2$ $4+4$	$4+4$	$4+4$

This conjecture includes as a special case the already established connection between membranes and $d=3$ extended super-YM theories. It is motivated by the following observation. If one asks for which number of supersymmetries and in which dimensions one can have a supersymmetric extension of a $(p-1)$ th rank antisymmetric tensor gauge theory, the answer is given precisely by the above table (given that field theories related to each other by dimensional reduction are considered equivalent). Of course, the known supersymmetric antisymmetric tensor gauge theories have an *Abelian* gauge invariance. The exception is $p=2$ for which the extension to the non-Abelian super-Yang–Mills theories are known. For the conjecture to be true there would have to be an analogous non-Abelian extension of the supersymmetric antisymmetric tensor gauge theories. Moreover, this extension should have an *infinite dimensional* gauge group.

APPENDIX

Our conventions are based on those of the $d=11$ supermembrane spelled out in $[12]$. For example,

$$\bar{\theta} = \theta^\dagger \Gamma_0 \tag{A.1}$$

defines $\bar{\theta}$. In many cases it is necessary to restrict θ to be Majorana and/or Weyl. A Majorana spinor is defined to satisfy

$$\bar{\theta} = \theta^T C, \quad (\text{A.2})$$

where the charge conjugation matrix C is unitary, and may be chosen to be real (in which case θ will be either real or purely imaginary).

The spinor indices on the Γ -matrices will be positioned as follows:

$$(\Gamma^\mu)^\alpha{}_\beta. \quad (\text{A.3})$$

Since the matrices $C\Gamma^\mu$ are either symmetric or antisymmetric, depending on the dimension d of spacetime, it is convenient to position the indices on them as follows

$$(C\Gamma^\mu)_{\alpha\beta}. \quad (\text{A.4})$$

As a consequence of these conventions the charge conjugation matrix can be thought of as a "metric" for raising and lowering spinor indices. Further properties of C , and whether the Majorana condition (A.2) is possible, depend on d . For $p \geq 2$ there are a total of eight values of d to be considered, i.e., $d = 4, \dots, 11$, and from (2.13) we observe that each value occurs precisely once. We shall specify below the conventions for each of these cases. In order to include all $p = 1$ cases we shall also need conventions for $d = 3$.

For $d = 3, 4, 10$, and 11 we have Majorana spinors with

$$C^T = -C, \quad (C\Gamma^\mu)^T = (C\Gamma^\mu), \quad (\text{A.5})$$

but for $d = 10$ we have the additional chirality constraint

$$\Gamma_{11}\theta = \pm\theta, \quad \Gamma_{11} = \Gamma^0\Gamma^1 \dots \Gamma^9. \quad (\text{A.6})$$

In all these cases we can take θ and the Γ -matrices to be real.

For $d = 8, 9$ we have Majorana spinors with

$$C^T = C, \quad (C\Gamma^\mu)^T = (C\Gamma^\mu). \quad (\text{A.7})$$

For $d = 5, 6, 7$ we cannot impose a Majorana condition so that θ is intrinsically complex. However, in these cases a complex spinor is equivalent to a pair of complex spinors $\theta^A (A = 1, 2)$ satisfying the *symplectic Majorana* condition

$$(\bar{\theta})^A = (\theta^T C')^B \varepsilon_{BA}, \quad (\varepsilon_{AB} = -\varepsilon_{BA}, \varepsilon_{12} = 1), \quad (\text{A.8})$$

where C' is the usual charge conjugation matrix satisfying

$$C'^T = -C', \quad (C'\Gamma'^\mu)^T = -(C'\Gamma'^\mu), \quad (d = 5) \quad (\text{A.9})$$

or

$$C' = C', \quad (C'\Gamma'^\mu)^T = -(C'\Gamma'^\mu), \quad (d = 6, 7) \quad (\text{A.10})$$

and $\{\Gamma'^\mu\}$ are the usual Dirac gamma matrices. In addition, for $d=6$ we have the chirality constraint

$$\Gamma'_7 \theta = \pm \theta, \quad \Gamma'_7 = \Gamma'^0 \Gamma'^1 \dots \Gamma'^5. \quad (\text{A.11})$$

For the sake of uniformity, it is convenient to choose a new set of reducible Γ -matrices

$$\Gamma^\mu = \Gamma'^\mu \otimes i\sigma_2, \quad (\text{A.12})$$

and a new charge conjugation matrix

$$C = C' \otimes i\sigma_2. \quad (\text{A.13})$$

For $d=5, 6, 7$ the symplectic Majorana condition (A.8) now becomes the standard Majorana condition (A.2) with C satisfying (A.5) for $d=6, 7$ and (A.7) for $d=5$.

With these conventions the action (2.1) with the $(p+1)$ -form B given in (2.6) is uniformly valid *except* for $p=3$. In this case the matrix

$$\begin{aligned} \mathcal{M} &= \mathbb{1} \otimes i\sigma_2 & (d=6) \\ \Gamma_9 &= \Gamma^0 \Gamma^1 \dots \Gamma^7 & (d=8) \end{aligned} \quad (\text{A.14})$$

must be inserted into the Wess–Zumino term and the κ -transformation rule for θ , as shown in subsection 3 of this appendix.

The Euclidean γ matrices γ^I ($I=1, \dots, (d-2)$) can be chosen to be all real for $(d-2)=1, 2, 8, 9$, or all purely imaginary for $(d-2)=6, 7$. In the latter case we take $\alpha \rightarrow i\alpha$ in order that (4.11) hold for all cases. For $(d-2)=3, 5$, we can choose a *reducible* representation (deriving from the reducible representation of the Γ -matrices given above) such that for $(d-2)=3, 5$, the matrices γ^I are again all real. The remaining case $p=3$, $(d-2)=4$, is special because the transverse Lorentz group is $SO(4) \equiv SO(3) \times SO(3)$ and the chirality condition in $d=6$ implies that the spinor S transforms according to the $(2, 1)$ representation, and we write it as S^A ($A=1, 2$). We can then write X^I , which transforms according to the $(2, 2)$ representation as $X^{AA'}$ satisfying the reality condition

$$(X^{AA'})^* = \varepsilon_{AB} \varepsilon_{A'B'} X^{BB'} \equiv X_{AA'}. \quad (\text{A.15})$$

We are using here the $SU(2)$ spinor convention

$$V_A \equiv V^B \varepsilon_{BA}, \quad U^B \equiv \varepsilon^{BA} U_A \quad (\text{A.16})$$

(where $\varepsilon^{01} = \varepsilon_{01} = 1$), and similarly for primed indices. The action and transformation rules in this notation for the $p=3$, $(d-2)=4$, case are given in subsection 3. In the same subsection is given the insertion of the matrix

$$\gamma_7 = \gamma^1 \dots \gamma^6 \quad (\text{A.17})$$

required for $p = 3$, $(d - 2) = 6$, in the action and transformation rules. With these conventions, for all p , the light-cone action follows from Eq. (4.1) and the light-cone symmetries are given by Eq. (4.11) and the definitions (3.9) and (3.33).

1. *Superstrings* ($d = 3, 4, 6, 10$)

Covariant:

$$\mathcal{L} = -\frac{1}{2}\sqrt{-g}g^{\bar{ij}}\Pi_i^\mu\Pi_{j\mu} - i\varepsilon^{\bar{ij}}\partial_i X^\mu\bar{\theta}\Gamma_\mu\partial_j\theta$$

$$\Gamma = \frac{1}{2!\sqrt{-g}}\varepsilon^{\bar{ij}}\Pi_i^\mu\Pi_j^\nu\Gamma_{\mu\nu}\left|\frac{1}{2}g^{\bar{ij}}\Pi_i^\mu\Pi_{j\mu}\right.$$

For $N = 2$ superstrings

$$\Pi_i^\mu = \partial_i X^\mu - i(\bar{\theta}^1\Gamma^\mu\partial_i\theta^1 + \bar{\theta}^2\Gamma^\mu\partial_i\theta^2)$$

and the $W - Z$ term is

$$-i\varepsilon^{\bar{ij}}\partial_i X^\mu(\bar{\theta}^1\Gamma_\mu\partial_j\theta^1 - \bar{\theta}^2\Gamma_\mu\partial_j\theta^2).$$

Light-cone:

$$\mathcal{L} = \frac{1}{2}(\partial_0 X^I\partial_0 X^I - \partial_1 X^I\partial_1 X^I) + \frac{i}{2}\bar{S}\partial_0 S - \frac{i}{2}\bar{S}\partial_1 S$$

$$\delta X^I = -2i\bar{S}\gamma^I\alpha$$

$$\delta S = -2\partial_0 X^I\gamma^I\alpha - 2\partial_1 X^I\gamma^I\alpha.$$

2. *Supermembranes* ($d = 4, 5, 7, 11$)

Covariant:

$$\mathcal{L} = -\frac{1}{2}\sqrt{-g}g^{\bar{ij}}\Pi_i^\mu\Pi_{j\mu} + \frac{1}{2}\sqrt{-g} - \frac{i}{2}\varepsilon^{\bar{ijk}}\bar{\theta}\Gamma_{\mu\nu}\partial_i\theta\left(\Pi_j^\mu\partial_k X^\nu - \frac{1}{3}\bar{\theta}\Gamma^\mu\partial_j\theta\bar{\theta}\Gamma^\nu\partial_k\theta\right)$$

$$\Gamma = \frac{1}{3!\sqrt{-g}}\varepsilon^{\bar{ijk}}\Pi_i^\mu\Pi_j^\nu\Pi_k^\rho\Gamma_{\mu\nu\rho}.$$

Light-cone:

$$\mathcal{L} = \frac{1}{2}\mathcal{D}_0 X^I\mathcal{D}_0 X^I - \frac{1}{2}\det\partial_a X^I\partial_b X^I + i\bar{S}\mathcal{D}_0 S$$

$$- \frac{1}{2}\varepsilon^{ab}\partial_a X^I\bar{S}\gamma^I\partial_b S,$$

$$\delta X^I = 2i\bar{\alpha}\gamma^I S$$

$$\delta S = -2\mathcal{D}_0 X^I\gamma_I\alpha - \varepsilon^{ab}\partial_a X^I\partial_b X^J\gamma_{IJ}\alpha$$

$$\delta\omega = -2i\bar{S}\alpha.$$

3. *Superlumps* ($d = 6, 8$)Covariant in $d = 6$:

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{2}\sqrt{-g}g^{\bar{ij}}\Pi_i^\mu\Pi_j^\mu\sqrt{-g} \\
& + \frac{1}{4!}\varepsilon^{i_1\cdots i_4}\bar{\theta}\Gamma_{v_1\cdots v_3}\mathcal{M}\partial_{i_4}\theta(4\Pi_{i_1}^{v_1}\cdots\Pi_{i_3}^{v_3}+6i\Pi_{i_1}^{v_1}\Pi_{i_2}^{v_2}\bar{\theta}\Gamma^{v_3}\partial_{i_3}\theta \\
& - 4\Pi_{i_1}^{v_1}\bar{\theta}\Gamma^{v_2}\partial_{i_2}\theta\bar{\theta}\Gamma^{v_3}\partial_{i_3}\theta - i\bar{\theta}\Gamma^{v_1}\partial_{i_1}\theta\bar{\theta}\Gamma^{v_2}\partial_{i_2}\theta\bar{\theta}\Gamma^{v_3}\partial_{i_3}\theta) \\
\Gamma = & \frac{-i}{4!\sqrt{-g}}\varepsilon^{ijkl}\Pi_i^\mu\Pi_j^\nu\Pi_k^p\Pi_l^\sigma\Gamma_{\mu\nu p\sigma}, \quad \delta\theta = \mathcal{M}(1+\Gamma)\kappa
\end{aligned}$$

Light-cone in $d = 6$:

$$\begin{aligned}
\mathcal{L} = & \frac{1}{2}\mathcal{D}_0X^{AA'}\mathcal{D}_0X_{AA'} - \frac{1}{3}\det\partial_aX^{AA'}\partial_bX_{AA'} + \frac{i}{2}S^A\mathcal{D}_0S_A \\
& + i\varepsilon^{abc}\partial_aX_{AA'}\partial_bX_B^{A'}S^A\partial_cS^B \\
\delta X^{AA'} = & 2i\alpha^{A'}S^A \\
\delta S^A = & 2\mathcal{D}_0X^{AA'}\alpha_{A'} - \frac{4}{3}\varepsilon^{abc}\partial_aX^A{}_{C'}\partial_bX_D^{C'}\partial_cX^{DD'}\alpha_{D'} \\
\delta\omega^{ab} = & -4i\varepsilon^{abc}\partial_cX^{AA'}\alpha_{A'}S_A.
\end{aligned}$$

Covariant in $d = 8$:

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{2}\sqrt{-g}g^{\bar{ij}}\Pi_i^\mu\Pi_{j\mu} + \sqrt{-g} \\
& + \frac{1}{4!}\varepsilon^{i_1\cdots i_4}\bar{\theta}\Gamma_{v_1\cdots v_3}\gamma_9\partial_{i_4}\theta(4\Pi_{i_1}^{v_1}\cdots\Pi_{i_3}^{v_3}+6i\Pi_{i_1}^{v_1}\Pi_{i_2}^{v_2}\bar{\theta}\Gamma^{v_3}\partial_{i_3}\theta \\
& - 4\Pi_{i_1}^{v_1}\bar{\theta}\Gamma^{v_2}\partial_{i_2}\theta\bar{\theta}\Gamma^{v_3}\partial_{i_3}\theta - i\bar{\theta}\Gamma^{v_1}\partial_{i_1}\theta\bar{\theta}\Gamma^{v_2}\partial_{i_2}\theta\bar{\theta}\Gamma^{v_3}\partial_{i_3}\theta) \\
\Gamma = & \frac{1}{3!\sqrt{-g}}\varepsilon^{ijkl}\Pi_i^\mu\Pi_j^\nu\Pi_k^p\Pi_l^\sigma\Gamma_{\mu\nu p\sigma}, \quad \delta\theta = \gamma_9(1+\Gamma)\kappa.
\end{aligned}$$

Light-cone in $d = 8$:

$$\begin{aligned}
\mathcal{L} = & \frac{1}{2}\mathcal{D}_0X^I\mathcal{D}_0X_I - \frac{1}{2}\det\partial_aX^I\partial_bX_I + \frac{i}{2}\bar{S}\mathcal{D}_0S \\
& + \frac{1}{4}\varepsilon^{abc}\partial_aX^I\partial_bX^J\bar{S}\gamma_{IJ}\gamma_7\partial_cS
\end{aligned}$$

$$\delta X^I = 2i\bar{\alpha}\gamma^I S$$

$$\delta S = -2\mathcal{D}_0 X^I \gamma_I \alpha - \frac{i}{3} \varepsilon^{abc} \partial_a X^I \partial_b X^J \partial_c X^K \gamma_{IJK} \gamma_7 \alpha$$

$$\delta\omega^{ab} = 2\varepsilon^{abc} \partial_c X^I \bar{\alpha} \gamma_I \gamma_7 S.$$

4. Super 4-Brane ($d=9$)

Covariant:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \sqrt{-g} g^{ij} \Pi_i^\mu \Pi_{j\mu} + \frac{3}{2} \sqrt{-g} \\ & + \frac{1}{5!} \varepsilon^{i_1 \dots i_5} \bar{\theta} \Gamma_{v_1 \dots v_4} \partial_{i_5} \theta (5 \Pi_{i_1}^{v_1} \dots \Pi_{i_4}^{v_4} + 10i \Pi_{i_1}^{v_1} \dots \Pi_{i_3}^{v_3} \bar{\theta} \Gamma^{v_4} \partial_{i_4} \theta \\ & - 10 \Pi_{i_1}^{v_1} \Pi_{i_2}^{v_2} \bar{\theta} \Gamma^{v_3} \partial_{i_3} \theta \bar{\theta} \Gamma^{v_4} \partial_{i_4} \theta \\ & - 5i \Pi_{i_1}^{v_1} \bar{\theta} \Gamma^{v_2} \partial_{i_2} \theta \dots \bar{\theta} \Gamma^{v_4} \partial_{i_4} \theta + \bar{\theta} \Gamma^{v_1} \partial_{i_1} \theta \dots \bar{\theta} \Gamma^{v_4} \partial_{i_4} \theta) \\ \Gamma = & \frac{-i}{5! \sqrt{-g}} \varepsilon^{ijklm} \Pi_i^\mu \Pi_j^\nu \Pi_j^\rho \Pi_l^\sigma \Pi_m^\lambda \Gamma_{\mu\nu\rho\sigma\lambda}. \end{aligned}$$

Light-cone:

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \mathcal{D}_0 X^I \mathcal{D}_0 X^I - \frac{1}{2} \det \partial_a X^I \partial_b X^I + \frac{i}{2} \bar{S} \mathcal{D}_0 S \\ & - \frac{1}{12} \varepsilon^{abcd} \partial_a X^I \partial_b X^J \partial_c X^K \bar{S} \gamma_{IJK} \partial_d S \\ \delta X^I = & 2i\bar{\alpha}\gamma^I S \\ \delta S = & -2\mathcal{D}_0 X^I \gamma_I \alpha + \frac{i}{12} \varepsilon^{abcd} \partial_a X^I \partial_b X^J \partial_c X^K \partial_d X^L \gamma_{IJKL} \alpha \\ \delta\omega^{ab} = & -\varepsilon^{abcd} \partial_c X^I \partial_d X^J \bar{\alpha} \gamma_{IJ} S. \end{aligned}$$

5. Super 5-Brane ($d=10$)

Covariant:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \sqrt{-g} g^{ij} \Pi_i^\mu \Pi_{j\mu} + 2\sqrt{-g} \\ & + \frac{1}{6!} \varepsilon^{i_1 \dots i_6} \bar{\theta} \Gamma_{v_1 \dots v_5} \partial_{i_6} \theta (6 \Pi_{i_1}^{v_1} \dots \Pi_{i_5}^{v_5} + 15i \Pi_{i_1}^{v_1} \dots \Pi_{i_4}^{v_4} \bar{\theta} \Gamma^{v_5} \partial_{i_5} \theta \\ & - 20 \Pi_{i_1}^{v_1} \dots \Pi_{i_3}^{v_3} \bar{\theta} \Gamma^{v_4} \partial_{i_4} \theta \bar{\theta} \Gamma^{v_5} \partial_{i_5} \theta - 15i \Pi_{i_1}^{v_1} \Pi_{i_2}^{v_2} \bar{\theta} \Gamma^{v_3} \partial_{i_3} \theta \dots \bar{\theta} \Gamma^{v_5} \partial_{i_5} \theta \\ & + 6 \Pi_{i_1}^{v_1} \bar{\theta} \Gamma^{v_2} \partial_{i_2} \theta \dots \bar{\theta} \Gamma^{v_5} \partial_{i_5} \theta + i \bar{\theta} \Gamma^{v_1} \partial_{i_1} \theta \dots \bar{\theta} \Gamma^{v_5} \partial_{i_5} \theta) \\ \Gamma = & \frac{1}{6! \sqrt{-g}} \varepsilon^{i_1 \dots i_6} \Pi_{i_1}^{v_1} \dots \Pi_{i_6}^{v_6} \Gamma_{v_1 \dots v_6}. \end{aligned}$$

Light-cone:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2} \mathcal{D}_0 X^I \mathcal{D}_0 X^I - \frac{1}{2} \det \partial_a X^I \partial_b X^I + \frac{i}{2} \bar{S} \mathcal{D}_0 S \\ &\quad + \frac{i}{48} \varepsilon^{abcde} \partial_a X^I \partial_b X^J \partial_c X^K \partial_d X^L \bar{S} \gamma_{IJKL} \partial_e S \\ \delta X^I &= 2i \bar{\alpha} \gamma^I S \\ \delta S &= -2 \mathcal{D}_0 X^I \gamma_I \alpha + \frac{2}{5!} \varepsilon^{abcde} \partial_a X^I \partial_b X^J \partial_c X^K \partial_d X^L \partial_e X^M \gamma_{IJKLM} \alpha \\ \delta \omega^{ab} &= \frac{i}{3} \varepsilon^{abcde} \partial_c X^I \partial_d X^J \partial_e X^K \bar{\alpha} \gamma_{IJK} S.\end{aligned}$$

ACKNOWLEDGMENTS

The authors would like to thank A. Achúcarro, S. Gandhi, C. Hull, C. Pope and K. Stelle for useful discussions. One of the authors (P.K.T.) would like to thank Professor Abdus Salam and the International Atomic Energy Agency for hospitality at the International Centre for Theoretical Physics, Trieste. Another of the authors (E.S.) is grateful for the hospitality of the members of the physics department of Imperial College, where part of this work was done.

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